Geometric Analysis, Submanifolds and Geometry of PDE's

Politecnico di Torino, September 9-13 2019

Book of abstracts

Christian Bär (Universität Potsdam), *Counterintuitive approximations*

Abstract: The Nash-Kuiper embedding theorem is a prototypical example of a counterintuitive approximation result: any short embedding of a Riemannian manifold into Euclidean space can be approximated by *isometric* ones. As a consequence, any surface can be isometrically C^1-embedded into an arbitrarily small ball in $\mathbb{R}^3$. For C^2-embeddings this is impossible due to curvature restrictions.

We will present a general result which will allow for approximations by functions satisfying strongly overdetermined equations on open dense subsets. This will be illustrated by three examples: real functions, embeddings of surfaces, and abstract Riemannian metrics on manifolds.

Our method is based on "weak flexibility", a concept introduced by Gromov in 1986. This is joint work with Bernhard Hanke.

Olivier Biquard (ÉNS Paris), *Renormalized volume for ALE Ricci-flat 4-manifolds*

Abstract: We introduce a renormalized volume for ALE Ricci-flat 4-manifolds, and prove that it satisfies an inequality, with equality only in the case of a flat cone. Joint work with Hans-Joachim Hein.

Francesco Bonsante (Università di Pavia), *Unbounded convex sets in hyperbolic space*

Abstract: A classical theorem of Alexandrov states that any metric on the sphere with curvature bounded from below by -1 is realised as the boundary of a closed convex set in hyperbolic space, and this set is unique up to isometry. In the talk I will discuss generalisation of this result for unbounded convex sets, considering in particular convex sets whose ideal boundary is a Jordan curve. In this setting, it is natural to augment the notion of induced metric on the boundary to include a gluing map at infinity which records how the asymptotic geometry of the two surfaces compares near points of the ideal boundary. For a given gluing map, under some natural assumption on the regularity, I will discuss an existence result of a convex set which realises the gluing map and whose induced metric on the boundaries has constant curvature.

This is a joint work with Jeff Danciger, Sara Maloni, and Jean-Marc Schlenker.

Reto Buzano (QMUL London - Università di Torino), *Bubbling analysis for closed and free-boundary minimal hypersurfaces*

Abstract: We will discuss a variety of weak and strong compactness results for sequences of minimal hypersurfaces (with and without boundary) in compact Riemannian manifolds of dimension between 3 and 7. In particular, exploiting a precise bubbling analysis, we obtain new strong compactness theorems in dimension 3 that extend results of Choi-Schoen and Fraser-Li from ambient manifolds with positive Ricci curvature to manifolds with only positive scalar curvature under the additional assumption of an index bound. The presented results have been obtained in joint work with Lucas Ambrozio, Alessandro Carlotto, and Ben Sharp.

Gilles Carron (Université de Nantes), *Gromov-Hausdorff compactness within a conformal class*

Abstract: I will talk on a joint work (https://arxiv.org/abs/1810.05387) with Clara Aldana (University de Norte, Barranquilla, Colombia) and Samuel Tapie (University of Nantes, France). Let (M^n, g_0) be a a closed Riemannian manifold, I will describe what kind of compactness theorem can be obtained for conformal metric $g_f = e^{2f} g_0$ with unit volume and uniform $L^{n/2}$ bound on the scalar curvature.

Eleonora Di Nezza (Sorbonne Universitè), *Metric geometry of singularity types*

Abstract: (Quasi)-Plurisubharmonic functions are a key notion in complex geometry. The study of their singularity (in terms -for example- of integrability properties or smoothing procedures) is conceived to develop analytic techniques in order to solve problems in complex and algebraic geometry.

In this talk we study the space of all possible singularity types of quasi-plurisubharmonic functions and we introduce a natural (pseudo)-distance on it. As applications we present a stability result for complex Monge-Ampre equations with prescribed singularity and a semicontinuity result for multiplier ideal sheaves associated to singularity types. This is a joint work with T. Darvas and C. Lu.

Anna Fino (Università di Torino), *Closed G_2-structures and Laplacian flow*

Abstract: I will review known examples of compact 7-manifolds admitting a closed G_2-structure. Moreover, I will discuss some results on the behaviour of the Laplacian G_2-flow starting from a closed G_2-structure whose induced metric satisfies suitable extra conditions.

Alexander Lytchak (Universität Köln), *Infinite geodesics on convex surfaces*

Abstract: In the talk I will discuss the following result and related analytic and geometric questions: On the boundary of any convex body in the Euclidean space there exist many infinite local geodesics.

Andrea Malchiodi (Scuola Normale Superiore, Pisa), *Prescribing Morse scalar curvatures in high dimension*

Abstract: We consider the classical problem of prescribing the scalar curvature of a manifold via conformal deformation of the metric, dating back to works by Kazdan and Warner. This problem is mainly understood in low dimensions, where blow-ups of solutions are proven to be "isolated simple". We find natural conditions to guarantee this also in arbitrary dimensions, when the prescribed curvatures are Morse functions. As a consequence, we improve some pinching conditions in the literature and derive existence results of new type. This is joint work with M. Mayer.

Carlo Mantegazza (Università di Napoli Federico II), *Evolution by curvature of networks in the plane*

Abstract: I will present the state-of-the-art of the problem of the motion by curvature of a network of curves in the plane, discussing existence, uniqueness, singularity formation and asymptotic behavior of the flow.

Luciano Mari (Università di Torino), *On the $1/H$ flow via p-Laplace approximation under Ricci lower bounds*

Abstract: In this talk, we consider the existence problem for weak solutions of the Inverse Mean Curvature Flow on a complete manifold with only a Ricci lower bound. Solutions either issue from a point or from the boundary of a relatively compact open set. To prove their existence in the sense of Huisken-Ilmanen, we follow the strategy pioneered by R. Moser using approximation by p-Laplacian kernels. In particular, we prove new and sharp gradient estimates for the kernel of the p-Laplacian on M via the study of the fake distance associated to it. We address the compactness of the flowing hypersurfaces, and time permitting some monotonicity formulas in the spirit of Geroch's one.
This is joint work with M. Rigoli and A. G. Setti.

Vladimir Matveev (Friedrich-Schiller-Universität Jena), *Nijenhuis Geometry: singularities and global issues*

Abstract: Abstract. The talk is devoted to Nijenhuis operators, i.e., fields of endomorphisms with vanishing Nijenhuis torsion. This is possibly the simplest geometric condition and appeared independently many times in geometry, analysis and mathematical physics, in particular in the theory of projectively equivalent metrics and in the theory of integrable systems. The talk is based on a series of papers with A. Bolsinov and A. Konyaev) aiming at re-directing the research agenda in this area to "next level topics":

- Singular points: what does it mean for a point to be generic or singular in the context of Nijenhuis geometry? What singularities are non-degenerate? What singularities are stable? How do Nijenhuis operators behave near non-degenerate and stable singular points?
- Global properties: what restrictions on a Nijenhuis operator are imposed by compactness of the manifold? And conversely, what are topological obstructions on a manifold carrying a Nijenhuis operator with specific properties (e.g.with no singular points) ?

More details can be found in the list of "Open problems, questions and challenges in finite-dimensional integrable systems" (coauthored by Bolsinov, Matveev, Miranda, Tabachnikov), and also in two already pre-published papers with title "Nijenhuis Geometry", where we demonstrate that this research program is realistic by proving a series of new, not at all obvious, results.

Pablo Mira (Universidad Politécnica de Cartagena), *Weingarten spheres in homogeneous three-manifolds*

Abstract: A classical theorem by H. Hopf shows that any compact constant mean curvature (CMC) surface of genus zero immersed in the Euclidean 3-space is a round sphere. This was generalized by Abresch and Rosenberg: any compact CMC surface of genus zero immersed in a rotationally symmetric homogeneous three-manifold is a rotational sphere. In this talk we will present some wide generalizations of these results. In particular, we will show that any compact special Weingarten surface of genus zero immersed in the product space $\mathbb{H}^2 \times \mathbb{R}$ is a rotational sphere, and we will extend the Abresch-Rosenberg classification to the case of surfaces with constant positive extrinsic curvature. Joint work with J.A. Gálvez.

Andrea Mondino (University of Warwick), *Optimal transport and quantitative geometric inequalities*

Abstract: The goal of the talk is to discuss a quantitative version of the Levy-Gromov isoperimetric inequality (joint with Cavalletti and Maggi) as well as other geometric/functional inequalities (joint with Cavalletti and Semola). Given a closed Riemannian manifold with strictly positive Ricci tensor, one estimates the measure of the symmetric difference of a set with a metric ball with the deficit in the Levy-Gromov inequality. The results are obtained via a quantitative analysis based on the localisation method via L^1-optimal transport.

Stefano Montaldo (Università di Cagliari), *Higher order energy functionals*

Abstract: The study of higher order energy functionals was proposed by Eells-Sampson in 1965 and by Eells-Lemaire in 1983. These functionals provide a natural generalization of the classical energy functional. More precisely, Eells and Sampson proposed the so-called ES-r-energy functionals $E_r^{ES}(\varphi) = (1/2)\int_M |(d^* + d)^r(\varphi)|^2\, dV$, where $\varphi : M \to N$ is a map between two Riemannian manifolds. In the initial part of this lecture we shall clarify some relevant issues about the definition of an ES-r-harmonic map, i.e, a critical point of $E_r^{ES}(\varphi)$. This is important because in the literature other higher order energy functionals have been studied by several authors and consequently some recent examples need to be discussed and extended. Next, we shall give the Euler-Lagrange system of equations for $E_r^{ES}(\varphi)$ for $r = 4$. We apply the above system to the study of maps into space forms and to rotationally symmetric maps.
This is a joint work with V. Branding, C. Oniciuc and A. Ratto.

Giovanni Moreno (University of Warsaw), *Invariant second-order PDEs on homogeneous contact manifolds*

Abstract: I will explain how to solve the following problem: for any simple complex Lie group G, construct a second-order PDE whose group of symmetries is precisely G. The very problem leads, in a natural manner, to considering G-homogeneous contact manifolds and their prolongations: the latter are the varieties of integral elements of the (homogeneous) contact structure, understood as an exterior differential system. Having recast the problem within the abstract fiber of the prolongation, the issue boils down to finding the one-dimensional representations of the residual action of G; our method will also ensure a certain "minimality" of the so-obtained PDEs.

Barbara Nelli (Università dell'Aquila), *The topology of constant mean curvature surfaces with convex boundary*

Abstract: What is the topology of a constant mean curvature surface with boundary a convex curve in the plane?

We give an overview of what is known about this problem in space forms. Moreover, adding some simple hypothesis, we answer to the previous question when the ambient manifold is $\mathbb{H}^2 \times \mathbb{R}$. Here $\mathbb{H}^2$ is the hyperbolic plane. This is a joint work with V. Moraru

Lorenzo Nicolodi (Università di Parma), *Pseudospherical Helicoids in the Large*

Abstract: It is known that the class of traveling wave solutions of the sine-Gordon equation is in 1-1 correspondence with the class of (necessarily singular) pseudospherical surfaces in Euclidean space with screw-motion symmetry: the pseudospherical helicoids. We illustrate our solution to the problem of explicitly describing all pseudospherical helicoids posed by A. Popov in [Lobachevsky Geometry and Modern Nonlinear Problems, Birkhäuser, Cham, 2014]. As an application, countably many continuous families of topologically embedded pseudospherical helicoids are constructed. This is joint work with Emilio Musso.

Pawel Nurowski (Polish Academy of Sciences), *Rolling without slipping or twisting and G_2*

Abstract: I will discuss conditions under which two surfaces rolling on each other without slipping or twisting define a velocity distribution having split real form of the exceptional simple Lie group G_2 as a symmetry. I will provide the only known examples of surfaces having this property.

Joaquín Pérez (Universidad de Granada), *Constant mean curvature surfaces of finite index*

Abstract: Let X be a complete Riemannian 3-manifold with positive injectivity radius and bounded sectional curvature. We will show that complete immersed surfaces M in X with constant mean curvature H between 0 and H_0 and stability index at most I (here H_0 is any positive number and I a nonnegative integer) have their interesting geometry localized around at most I points where the norm of the second fundamental form of the surface takes on large local maximum values. We will obtain as consequence that when M is closed, its area grows linearly with its genus. This is on going joint work with Bill Meeks.

Stefano Pigola (Università dell'Insubria), *Subsolutions at infinity of nonlinear PDEs: decay, compact support and global Sobolev regularity*

Abstract: In this talk I will present some recent results concerning the global behaviour of positive and bounded solutions u of differential inequalities of the form $\Delta_p u \geq f(u)$ on the exterior domain of a complete manifold. I shall discuss how the geometry of the manifold forces u to vanish at infinity and, hence, to have compact support. Key tools are a nonlinear version of the so called Feller property and a global comparison result. This latter, in turn, is related

to a new characterization of the stochastic completeness in terms of the Sobolev space $W^{1,p}$. It is a joint work with Davide Bianchi and Alberto G. Setti.

Giuseppe Pipoli (Università dell'Aquila), *Invariant translators of the Heisenberg group*

Abstract: Translating solutions to the mean curvature flow are special hypersurfaces that move under mean curvature flow preserving their shape and translating in a fixed direction. They have a crucial role in understanding the singularities of the flow and provide interesting explicit examples of solutions. In this talk we present the construction of infinitely many new translating surfaces in the Heisenberg group. We discuss similarities and differences with the analogous examples in the Euclidean space.

Tristan Rivière (ETH Zürich), *Immersed 2-spheres in $\mathbb{R}^3$: A Morse theoretic perspective*

Abstract: In their attempt to generalize Euler elastic theory of beams to flexible membranes, Sophie Germain and Siméon Poisson introduced, two centuries ago, a lagrangian which has now become a mathematical object whose study goes a way beyond the mechanics of bent surfaces. The so called Willmore Lagrangian is a functional that shows up in many areas of sciences such as conformal geometry, general relativity, cell biology, optics...etc.

We will try to shed some lights on the universality of this Lagrangian. One remarkable fact is a quantization phenomenon of the Willmore critical spherical membranes which happen to have all integer valued energy.

We shall then present the project of using the Willmore energy as a Morse function for studying the fascinating space of immersed 2-spheres in the euclidean 3 space and relate topological obstructions in this space to integer values and minimal surfaces.

Magdalena Rodríguez (Universidad de Granada), *Constant mean curvature surfaces in $\mathbb{H}^2\times\mathbb{R}$*

Abstract: It is well-known that there exists minimal surfaces in horizontal slabs of $\mathbb{H}^2 \times \mathbb{R}$. In fact Nelli and Rosenberg proved that, given any Jordan curve c at infinity projecting $1:1$ to the boundary at infinity of $\mathbb{H}^2$, there exists an entire minimal graph whose asymptotic boundary is c. We also know that for any $H > 1/2$, there exist spheres with constant mean curvature H; in particular, there are examples with constant mean curvature H in horizontal slabs. We will prove that this is not possible when $0 < H \leq 1/2$. This is a joint work with Laurent Hauswirth and Ana Menezes.

Dietmar Salamon (ETH Zürich), *Moment maps in symplectic and Kaehler geometry*

Abstract: TBA.

Alessandro Savo (Università di Roma "La Sapienza"), *Morse index and topology of minimal hypersurfaces and self-shrinkers*

Abstract: It is well-known that a hypersurface of a Riemannian manifold is critical for the area functional (that is, the first variation of area is zero) if and only if the hypersurface is minimal. The second variation of area is a quadratic form whose number of negative eigenvalues is often called the Morse (or stability) index of the given minimal immersion: it measures, roughly speaking, the number of independent directions along which the area is decreasing. It is reasonable to expect that (at least in positive ambient curvature) a complicated topology will imply high instability. In the talk we review some of the main results supporting this principle and discuss a recent result for weighted minimal hypersurfaces.

Carlo Sinestrari (Università di Roma "Tor Vergata"), *Curvature flows and isoperimetric inequalities*

Abstract: We consider the evolution of a hypersurface in Euclidean space with speed given by a function of the principal curvatures. The analysis of these flows has often connections with the

study of geometric inequalities, such as the Alexandrov-Fenchel type ones relating the mixed volumes in convex geometry. On one hand, in fact, the monotonicity of a suitable isoperimetric ratio has allowed to prove convergence to a spherical profile for certain classes of geometric flows. On the other hand, similar monotonicity properties have allowed to provide generalizations, or alternative proofs, of some classical isoperimetric inequalities. In this talk I will give a survey of some recent results of this kind.

Luigi Vezzoni (Università di Torino), *The quantitative Alexandrov Theorem*

Abstract: The celebrated Alexandrov theorem asserts that the distance spheres are the only closed constant mean curvature hypersurfaces embedded in space forms (assuming the hypersurfaces embedded in the hemisphere in the elliptic case). The talk mainly focuses on the following quantitive version of the Alexandrov theorem:

Theorem [Ciraolo -Roncoroni- V]**.** *Let S be an n-dimensional, C^2-regular, connected, closed hypersurface embedded in a space form (S embedded in a hemisphere in the elliptic case). There exist constants ϵ, $C > 0$ such that if*

$$\mathrm{osc}(H) \leq \epsilon,$$

then there are two concentric balls B_{r_i} and B_{r_e} such that

$$S \subset \overline{B}_{r_e} \setminus B_{r_i},$$

and

$$r_e - r_i \leq C\mathrm{osc}(H).$$

The constants ϵ and C depend only on n and upper bounds on the C^2-regularity and the area of S.

The proof of the theorem makes use of a quantitive study of the method of the moving planes and the result implies a new pinching theorem for hypersurfaces. Moreover the results generalizes to other curvature operators and is optimal in a sense that it will be specified in the talk.

Guofang Wei (University of California, Santa Barbara), *Fundamental Gap Estimate on Convex Domains of Sphere*

Abstract: In their celebrated work, B. Andrews and J. Clutterbuck proved the fundamental gap (the difference between the first two eigenvalues) conjecture for convex domains in the Euclidean space and conjectured similar results holds for spaces with constant sectional curvature. In several joint works with S. Seto, L. Wang; C. He; and X. Dai, S. Seto, we prove the conjecture for the sphere. Namely for any strictly convex domain in the unit S^n sphere, the gap is $\geq 3\frac{\pi^2}{D^2}$. As in B. Andrews and J. Clutterbuck's work, the key is to prove a super log-concavity of the first eigenfunction.